PHYSICAL REVIEW B **81**, 041305(R) (2010)

Quantum dot in a two-dimensional topological insulator: The two-channel Kondo fixed point

K. T. Law,[1,2] C. Y. Seng,[3] Patrick A. Lee,[2] and T. K. Ng[3]
[1]*Institute for Advanced Study, Hong Kong University of Science and Technology, Hong Kong, People's Republic of China*
[2]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[3]*Department of Physics, Hong Kong University of Science and Technology, Hong Kong, People's Republic of China*
(Received 14 December 2009; published 14 January 2010)

A quantum dot coupled to two helical edge states of a two-dimensional topological insulator through electron tunnelings is studied. We show that if the electron interactions on the edge states are repulsive, with Luttinger liquid parameter $K<1$, the system reaches a stable two-channel Kondo fixed point at low temperatures. This is in contrast to the Luttinger liquid leads case in which $K<1/2$ is needed. This two-channel fixed point is described by a boundary sine-Gordon Hamiltonian with a K dependent boundary term. The impurity entropy, the impurity specific heat and the conductance are calculated.

DOI: 10.1103/PhysRevB.81.041305 PACS number(s): 73.63.Kv, 71.10.Pm, 72.15.Qm, 72.25.Hg

The two-channel Kondo model has been under intense theoretical study[1] after the seminal work of Noziere and Blandin,[2] in which they pointed out that a non-Fermi liquid fixed point exists for the two-channel Kondo model with noninteracting electrons. Recently, the authors of Ref. 3 demonstrated experimentally that a quantum dot coupled to an infinite reservoir with noninteracting electrons and a finite reservoir with interacting electrons may display two-channel Kondo effect. The key idea is that Coulomb blockade suppresses the exchange of electrons between the finite reservoir and the infinite reservoir.[4] Consequently, the two reservoirs couple to the quantum dot independently.

Indeed, using electron-electron repulsion to suppress interreservoir tunnelings was suggested earlier by Fabrizio and Gogolin, in the context of a quantum dot coupled to two Luttinger liquid leads.[5–7] They show that the two-channel Kondo fixed point can be reached when the repulsive interactions of the electrons in the leads are strong enough, i.e., with Luttinger liquid parameter $K<1/2$.

Recently, a class of materials called topological insulators were first theoretically proposed and then experimentally fabricated.[8–12] Two-dimensional topological insulators have gapless helical edge states despite the presence of a bulk gap.[8] The edge states are called helical because the directions of the spins and the momentum of the electrons are tied together.[13] The one-channel Kondo effect of helical edge states was first studied in Ref. 14 as a truncated model of Kondo effect in Luttinger liquids. A more detailed recent study is carried out in Ref. 15.

In this work, we study a quantum dot coupled to two helical edge states as depicted in Fig. 1(a). If the electrons on the edge states are noninteracting, it is well known that the system can be described by a one-channel Kondo Hamiltonian.[16,17] However, we show that a weak repulsive interaction, with $K<1$ is enough to drive the system to the two-channel Kondo fixed point. This is in sharp contrast to the case of Luttinger liquid leads in which $K<1/2$ is needed.[5] This two-channel fixed point is described by a boundary sine-Gordon Hamiltonian with a K dependent boundary term. We study the ground state, thermaldynamic and transport properties near this two-channel fixed point. More specifically, we show that the impurity entropy at zero temperature is $\ln\sqrt{2K}$, the impurity specific-heat $C_{imp}\propto T^{2/K-2}$ when $2/3<K<1$, and $C_{imp}\propto T$ when $K<2/3$, and the linear conductance across the two helical edges through the quantum dot has nontrivial temperature dependence as a result of the renormalization group flow.

The schematic diagram of our model is depicted in Fig. 1(a). Two helical edge states of an two-dimensional (2D) topological insulator are brought close to each other at a tunneling junction. A quantum dot is placed at the middle of the junction and coupled to the two edge states through electron tunnelings. The system is described by an Anderson Hamiltonian $H_A=H_0+H_i+H_d+H_t$ where

$$H_0=\sum_{i=1,2}-iv_F\int[\psi^\dagger_{iR\uparrow}(x)\partial_x\psi_{iR\uparrow}(x)-\psi^\dagger_{iL\downarrow}(x)\partial_x\psi_{iL\downarrow}(x)]dx,$$

$$H_i=\sum_{i=1,2}\sum_{\alpha=R,L}\frac{g_4}{2}\int\psi^\dagger_{i\alpha\sigma}(x)\psi_{i\alpha\sigma}(x)\psi^\dagger_{i\alpha\sigma}(x)\psi_{i\alpha\sigma}(x)dx$$
$$+g_2\int\psi^\dagger_{iR\uparrow}(x)\psi_{iR\uparrow}(x)\psi^\dagger_{iL\downarrow}(x)\psi_{iL\downarrow}(x)dx$$

$$H_d=\sum_\sigma\epsilon d^\dagger_\sigma d_\sigma+Ud^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow,$$

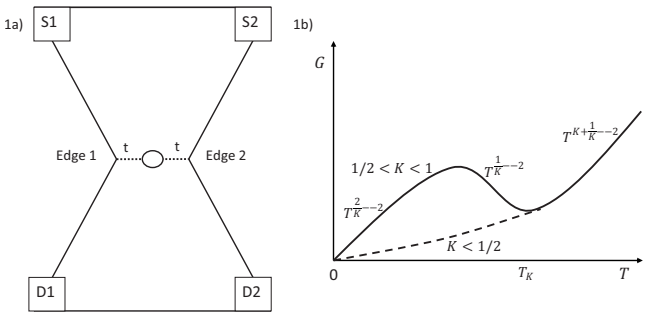

FIG. 1. (a) A quantum dot coupled to two helical edges of a 2D topological insulator through electron tunneling. S and D denote the source and drain, respectively. (b)The schematic picture of the linear conductance G from S1 to D2 versus temperature T. The solid line and the dashed line depict the $1/2<K<1$ and $K<1/2$ cases, respectively.

$$H_t = \sum_{i=1,2} t(d_\uparrow^\dagger \psi_{iR\uparrow} + d_\downarrow^\dagger \psi_{iL\downarrow}) + \text{h.c.}$$

H_0 and H_i denote the kinetic energy and the interaction of the helical edge states, respectively. From H_0, we see that the right/left moving electrons are spin up/down. H_d is the Hamiltonian of the quantum dot where ϵ is the single particle energy level at the dot and U is the on-site interaction energy. H_t describes the coupling between the dot and the edges and we have tuned to the channel symmetric point such that t is independent of the edge index. In the presence of spin-orbit coupling, spin ($\uparrow$ and $\downarrow$) denotes pseudospin of the edge modes. In the large U limit, one obtains the Kondo Hamiltonian $H=H_0+H_K$ through perturbation theory,[18] where

$$H_K = \sum_{i=1,2} J_1 \vec{S} \cdot \left(\psi_{i\alpha}^\dagger \cdot \frac{\sigma_{\alpha\beta}}{2} \cdot \psi_{i\beta} \right) + \sum_{i\neq j} J_2 \vec{S} \cdot \left(\psi_{i\alpha}^\dagger \cdot \frac{\sigma_{\alpha\beta}}{2} \cdot \psi_{j\beta} \right). \tag{1}$$

Initially, $J_1=J_2>0$ and without interaction, Eq. (1) reduces to a single channel Kondo problem.[16,17] Our goal is to show that J_2 normalizes to zero at low temperatures when $g_2>0$. Because of helicity, one of the R/L and the spin $\uparrow\downarrow$ indexes are redundant, we keep only one of them starting from Eq. (1).

It is important to note that, in general, there are extra backscattering terms of the form $\psi_{iR}^\dagger(0)\psi_{iL}(0)+\text{h.c.}$ caused by the quantum dot in the Hamiltonian H. The backscattering terms cut each edge into two separate parts at low temperatures if $K<1$ and render the geometry in Fig. 1(a) unstable. However, in our case, single particle back-scattering terms are forbidden because of time-reversal symmetry. Two-particle back scatterings preserve time-reversal symmetry but they are irrelevant so long as $K<1/4$.[13] Thus, our discussion below is valid for $K>1/4$.

By Abelian bosonization,[19,20] the electron operators can be written as: $\psi_{iR/L}=\frac{1}{\sqrt{2\pi a}} e^{\pm i(\sqrt{4\pi}\phi_{iR/L}(x)+k_F x)}$. We define bosonic fields $\phi_i(x)=\phi_{iL}(x)+\phi_{iR}(x)$ and $\theta_i(x)=\phi_{iL}(x)-\phi_{iR}(x)$. The dual fields satisfy the commutation relations $[\phi_i(x),\theta_j(x')] = \frac{-i}{2}\delta_{ij}\,\text{sgn}(x-x')$, where $\text{sgn}(x)=0$ when $x=0$. The symmetric and antisymmetric combinations of ϕ_i and θ_i are denoted as: $\phi_{s/a}=\frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2)$ and $\theta_{s/a}=\frac{1}{\sqrt{2}}(\theta_1 \pm \theta_2)$. In terms of $\phi_{s/a}$ and $\theta_{s/a}$, the above Hamiltonians can be written as: $H_0+H_i = \frac{v_F'}{2}\int \frac{1}{K}(\partial_x\phi_s)^2+K(\partial_x\theta_s)^2+\frac{1}{K}(\partial_x\phi_a)^2+K(\partial_x\theta_a)^2 dx$ and

$$\begin{aligned} H_K = &-\sqrt{\frac{2}{\pi}} J_1^z S_z \partial_x\theta_s(0) \\ &+ \frac{J_1^{xy}}{\pi a}[S^- e^{-i\sqrt{2\pi}\phi_s(0)} + S^+ e^{i\sqrt{2\pi}\phi_s(0)}]\cos[\sqrt{2\pi}\phi_a(0)] \\ &+ \frac{2J_2^z}{a\pi} S^z \sin[\sqrt{2\pi}\theta_a(0)]\sin[\sqrt{2\pi}\phi_a(0)] \\ &+ \frac{J_2^{xy}}{\pi a}[S^- e^{-i\sqrt{2\pi}\phi_s(0)} + S^+ e^{i\sqrt{2\pi}\phi_s(0)}]\cos[\sqrt{2\pi}\theta_a(0)]. \end{aligned}$$

Here, the Luttinger liquid parameter is defined as

$$K = \sqrt{\frac{1+g_4/2\pi v_F - g_2/2\pi v_F}{1+g_4/2\pi v_F + g_2/2\pi v_F}}$$

and $v_F'=v_F\sqrt{(1+g_4/2\pi v_F)^2-(g_2/2\pi v_F)^2}$.[20] The bosonic form of H_K is spin anisotropic, we denote the two components of J_i as $J_i^{z/xy}$, respectively. This is reasonable for our problem, because spin-orbit coupling breaks $SU(2)$ symmetry of the electrons on the edges. The spin-coupling terms acquire different scaling dimensions when $K\neq 1$ as we see below.

With the bosonic Hamiltonian $H=H_0+H_i+H_K$, we may calculate the scaling dimensions of the operators in H_K. At the vicinity of the fixed point where $J_i^z=J_i^{xy}=0$, the scaling dimensions of the J_1^z, J_1^{xy} terms are 1, and K, respectively. On the other hand, the scaling dimensions of the J_2^z and J_2^{xy} terms are $\frac{1}{2}(K+1/K)>1$. Thus, when $K<1$, the J_2 terms decrease but the J_1 terms grow when temperature is lowered.

In order to study the physics at the strong coupling regime when J_1 is of order 1, we use the Emery-Kivelson method by applying a unitary transformation $U=e^{i\sqrt{2\pi K}\phi_s(0)S^z}$ to H.[21] After the unitary transformation and the rescaling of the bosonic fields, we have

$$\begin{aligned} \tilde{H} &= U^\dagger H U \\ &= H_0 + \lambda S^z \partial_x\theta_s(0) \\ &\quad + \frac{2J_2^z}{a\pi} S^z \sin\left[\sqrt{\frac{2\pi}{K}}\theta_a(0)\right]\sin[\sqrt{2\pi K}\phi_a(0)] + (S^- + S^+) \\ &\quad \times \left\{ \frac{J_1^{xy}}{\pi a}\cos[\sqrt{2\pi K}\phi_a(0)] + \frac{J_2^{xy}}{\pi a}\cos\left[\sqrt{\frac{2\pi}{K}}\theta_a(0)\right] \right\}, \end{aligned} \tag{2}$$

where $\lambda=\sqrt{2\pi K}v_F'-\sqrt{\frac{2}{\pi K}}J_1^z$. At the vicinity of the point $\lambda = J_1^{xy}=J_2^z=J_2^{xy}=0$, we may calculate the scaling dimensions of the J_i terms as before. The J_1^{xy} and J_2^{xy} terms become more relevant because of the elimination of the $e^{\pm i\phi_s(0)}$ factors by the transformation. The J_1^{xy} term has scaling dimension $\frac{K}{2}$. It is relevant when $K<1$. The J_2^{xy} term has scaling dimension $\frac{1}{2K}$ and it is relevant when $1/2<K<1$.

A quantum dot coupled to two Luttinger liquid leads is described by a Hamiltonian H_L which is very similar to $\tilde{H}$ of Eq. (2). However, there is one crucial difference between H_L and $\tilde{H}$. In $\tilde{H}$, the two terms attached to the spin operators $S^+ + S^-$ are $\cos(\sqrt{2\pi K}\phi_a)$ and $\cos(\sqrt{2\pi/K}\theta_a)$, respectively. In H_L, the corresponding terms attached to $S^- + S^+$ are $\cos(\phi_\sigma)$ and $\cos(\phi_\rho)$, where ϕ_σ and ϕ_ρ are charge and spin bosonic fields, respectively, which are independent from each other.[5] In the present problem, ϕ_a and θ_a are dual fields of each other, pinning the value of the ϕ_a field increases the fluctuation of the θ_a field. We will see below that this is responsible for the fact that the two-channel Kondo fixed pointed is stable for $K<1$.

When $K<1$, the J_1^{xy} term is the most relevant term. At low temperatures, one expect the effective Hamiltonian of $\tilde{H}$ is described by the fixed point Hamiltonian:

TABLE I. Scaling dimensions of the operators (Op.) at different fixed points (FPs). ($J_2=0$ for all fixed points, only coefficients of corresponding operators shown.)

Op.\FPs	$J_1=0$	$\lambda=J_1^{xy}=0$	$\lambda=0$, $J_1^{xy}\to\infty$
$J_1^z(\delta\lambda)$	1	1	$1+1/2K$
J_1^{xy}	K	$K/2$	N/A
J_2^z	$\frac{1}{2}(K+1/K)$	$\frac{1}{2}(K+1/K)$	$1/K+1/2K$
J_2^{xy}	$\frac{1}{2}(K+1/K)$	$1/2K$	$1/K$

$$\tilde{H}_{eff}=H_0+\frac{2J_1^{xy}}{\pi a}S^x\cos[\sqrt{2\pi K}\phi_a(0)]. \tag{3}$$

To show that this fixed point is indeed stable, we need to calculate the scaling dimensions of the operators $J_2^{xy}S^x\cos[\sqrt{\frac{2\pi}{K}}\theta_a(0)]$, $\delta\lambda S_z\partial_x\theta_s(0)$, and $J_2^zS_z\sin[\sqrt{\frac{2\pi}{K}}\theta_a(0)]\sin[\sqrt{2\pi K}\phi_a(0)]$ at the vicinity of the fixed point. $\delta\lambda$ denotes the deviation from the Emery-Kivelson line. The results are given in the third column in Table I. We see that all these operators are irrelevant if $K<1$. Which confirms that Eq. (3) is the low-temperature effective Hamiltonian. Below we give some details of the calculation.

To calculate the scaling dimension of $J_2^{xy}S^x\cos[\sqrt{\frac{2\pi}{K}}\theta_a(0)]$, we note that S^x commutes with $\tilde{H}_{eff}$, so that we may set S^x to $\pm1/2$. As a result, one needs to calculate the scaling dimension of $\cos[\sqrt{\frac{2\pi}{K}}\theta_a(0)]$ with the Hamiltonian

$$\tilde{H}_{\pm}=H_0\pm\frac{J_1^{xy}}{\pi a}\cos[\sqrt{2\pi K}\phi_a(0)]. \tag{4}$$

Equation (4) has the form of the Hamiltonian of a spinless Luttinger liquid wire with an impurity backscattering term at point $x=0$. One may interpret the $\cos[\sqrt{2\pi K}\phi_a(0)]$ as an impurity back scattering term with Luttinger liquid parameter $K'=K/2$. At low temperatures, the "back scattering term" is relevant for $K'<1$. Thus, the $\cos[\sqrt{2\pi K}\phi_a(0)]$ term cuts the Luttinger liquid wire into two separate pieces at $x=0$.[22] As a result, the point $x=0$ can be regarded as a point at the boundary when $J_1^{xy}\to\infty$.

The effective Hamiltonian for half of the wire is $H_0'=\frac{v_F}{2}\int_0^L\{[\partial_x\phi_a(x)]^2+[\partial_x\theta_a(x)]^2\}dx$. Define chiral fields ϕ_L and ϕ_R by the relations $\theta_a(x)=\phi_L(x)-\phi_R(x)$ and $\phi_a(x)=\phi_L(x)+\phi_R(x)$ and apply the conditions $\phi_R(x)=\tilde{\phi}_R(x)$ and $\phi_L(x)=-\tilde{\phi}_R(-x)$, we have $H_0'=v_F\int_{-L}^L[\partial_x\tilde{\phi}_R(x)]^2dx$ and $e^{i\theta_a(x)\sqrt{2\pi/K}}=e^{-i[\tilde{\phi}_R(-x)+\tilde{\phi}_R(x)]\sqrt{2\pi/K}}$. Define free bosonic fields which are defined from $-L$ to L, $\tilde{\theta}(x)=\tilde{\phi}_L(x)-\tilde{\phi}_R(x)$ and $\tilde{\phi}(x)=\tilde{\phi}_L(x)+\tilde{\phi}_R(x)$, we have $e^{i\theta_a(x)\sqrt{2\pi/K}}=e^{-i(\tilde{\phi}(-x)-\tilde{\theta}(-x)+\tilde{\phi}(x)-\tilde{\theta}(x))\sqrt{\pi/2K}}$. At $x=0$, $e^{i\theta_a(0,t)\sqrt{2\pi/K}}=e^{-i(\tilde{\phi}(0,t)-\tilde{\theta}(0,t))\sqrt{2\pi/K}}$. As a result, the scaling dimension of $e^{i\theta_a(0,t)\sqrt{2\pi/K}}$ at the boundary can be easily calculated. It is $1/K$ instead of $1/2K$ as is the case in the bulk. This well known result[20] is crucial because the $J_2^{xy}S^x\cos[\sqrt{\frac{2\pi}{K}}\theta_a(0)]$ term is now irrelevant when $K<1$.

The calculations of the scaling dimension of the $S^z\partial_x\theta_s(0)$ term is similar to what have been done before in Ref. 23. Since θ_s is governed by a free Hamiltonian, we have $\langle S^z(\tau)\partial_x\theta_s(\tau)S^z(0)\partial_x\theta_s(0)\rangle=\langle S^z(\tau)S^z(0)\rangle\langle\partial_x\theta_s(\tau)\partial_x\theta_s(0)\rangle$

In order to calculate $\langle S^z(\tau)S^z(0)\rangle_{\tilde{H}_{eff}}$, we note that S^z is the sum of the raising and lowering operators of the eigenstates of S^x, $|\pm\rangle$. Thus, we have

$$\langle-|e^{\tilde{H}_{eff}\tau}S^z(0)e^{-\tilde{H}_{eff}\tau}S^z(0)|-\rangle=\langle-|e^{\tilde{H}_-\tau}W^\dagger(0)e^{-\tilde{H}_-\tau}W(0)|-\rangle$$
$$=\langle-|W^\dagger(\tau)W(0)|-\rangle_{\tilde{H}_-} \tag{5}$$

where W is a unitary transformation which satisfies $W^\dagger e^{-\tilde{H}_-\tau}W=e^{-\tilde{H}_+\tau}$.

Actually, we need to find a unitary operator W which transforms ϕ_a to $\phi_a+\sqrt{\frac{\pi}{2K}}$. Indeed, $W=e^{-i\sqrt{\pi/2K}[\theta_a(L)-\theta_a(-L)]}$, where $2L$ is the length of the system. In order to calculate $\langle-|W^\dagger(\tau)W(0)|-\rangle_{\tilde{H}_-}$, we come back to the spinless Luttinger liquid Hamiltonian interpretation of $\tilde{H}_-$.

Suppose we have a spinless Luttinger liquid wire with an impurity at $x=0$ and the left and right ends locate at $-L$ and L, respectively. At the strong coupling fixed point, the wire is cut into two separate parts by the impurity. However, the left half and the right half are described by free Hamiltonians H_{0L} and H_{0R}, respectively. As a result, we have

$$\langle W^\dagger(\tau)W(0)\rangle_{\tilde{H}_-}\approx\langle e^{i\sqrt{\pi/2K}[\theta_a(-L,\tau)-\theta(-L,0)]}\rangle_{H_{0L}}$$
$$\times\langle e^{i\sqrt{\pi/2K}[\theta_a(L,\tau)-\theta(L,0)]}\rangle_{H_{0R}}\propto\frac{1}{\tau^{1/K}}. \tag{6}$$

Since $\partial_x\theta_s(0)$ has a scaling dimension of 1, we conclude that the operator $S^z\partial_x\theta_s(0)$ is irrelevant with scaling dimension $1/2K+1$.

One may calculate the scaling dimension of the $J_2^zS_z\sin[\sqrt{\frac{2\pi}{K}}\theta_a(0)]\sin[\sqrt{2\pi K}\phi_a(0)]$ term in a similar way. This term has scaling dimension $\frac{1}{2K}+\frac{1}{K}$. It is irrelevant when $K<1$.

We have shown above that the two-channel fixed point is stable. At the vicinity of this fixed point, the system is described by $\tilde{H}_{eff}$ in Eq. (3). It is interesting to note that $\tilde{H}_{eff}$ can be regarded as a generalization of the fixed point Hamiltonian of the two-channel and four-channel Kondo non-Fermi liquid fixed-point Hamiltonians. The two-channel and four-channel cases are described by $\tilde{H}_{eff}$ with $K=1$ and $K=3/2$ respectively.[26] One of the most remarkable features of the multichannel Kondo effect is the existence of fractionally degenerate ground state. For example, the two-channel and four-channel Kondo models have residual entropy of $\frac{1}{2}\ln 2$ and $\frac{1}{2}\ln 3$, respectively.[24,25] This motivates us to study the residual entropy of $\tilde{H}_{eff}$ for $K<1$.

In order to calculate the residual entropy, we first calculate the partition function $Z=Tr\{\exp^{-\beta\tilde{H}_{eff}}\}$. Since S^x commutes with the $\tilde{H}_{eff}$, the states can be labeled as $|\pm\frac{1}{2},\phi_a\rangle$. We may write $Z=Z_+ + Z_-=Tr\{\exp^{-\beta\tilde{H}_+}\}+Tr\{\exp^{-\beta\tilde{H}_-}\}$. Remember that the integral of H_0 run from $-\infty$ to $+\infty$ if $L\to\infty$ is taken. However, we may define bosonic fields such that the positive axis is folded to the negative axis.[27] After folding, the effect

of the cosine term in $\tilde{H}_{\pm}$ is to introduce a Dirichlet boundary condition. The actual value of $\phi_a(0,\tau)$, whether it is pinned at $(2n+1)\pi/\sqrt{2\pi K}$ or $2n\pi/\sqrt{2\pi K}$, does not affect the partition function.[28] Thus, we have $Z_+=Z_-$ and $Z=2Z_+$. The impurity entropy of $\tilde{H}_+$ has been calculated in Ref. 27. It is $\ln g=\ln\sqrt{K/2}$, where g is the "ground-state degeneracy" discussed in Ref. 25. Together with the contribution from S^x, the total impurity entropy is

$$S=\ln\sqrt{2K}. \tag{7}$$

Substituting $K=1$ and $K=3/2$ into Eq. (7) reproduces the impurity entropy results, calculated by the Bethe ansatz and boundary conformal field theory, for the two-channel and four-channel Kondo effects with noninteracting electrons, respectively.[24,25]

We show above that the residual impurity entropy is determined by the boundary term. On the other hand, the thermodynamic properties are determined by the leading irrelevant operators (LIO) near the fixed point. Since the irrelevant operators have correlation functions of the form $\langle\langle O(\tau)O(0)\rangle\rangle=[\frac{\pi T}{\sin(\pi T\tau)}]^{2\Delta}$, the second-order correction of the free energy $\delta F^{(\lambda)}(T)=-\lambda^2\int_{\tau_0}^{\beta/2}d\tau[\frac{\pi T}{\sin(\pi T\tau)}]^{2\Delta}$ where τ_0 is the cut-off time.[23]

Near the strong coupling fixed point, the $\frac{J_2^{xy}}{\pi a}\cos[\sqrt{\frac{2\pi}{K}}\theta_a(0)]$ term with scaling dimension $1/K$ is the LIO if $1/2<K<1$. We have $\delta F\propto T^{2/K-1}+0(T^2)$, where the $T^{2/K-1}$ term is universal and independent of the cut-off time τ_0 and the $0(T^2)$ term has a τ_0 dependent coefficient. As a result, the impurity specific heat $C_{imp}\propto T^{2/K-2}$ if $2/3<K<1$. However, when $K<2/3$, the free energy is dominated by the $0(T^2)$ term and $C_{imp}\propto T$.

In the above calculations, we have assumed that the two helical edges have equal chemical potential. If a small bias is applied across the two edges, there should be a current flowing from one edge to the other. At temperatures much higher than the Kondo temperature, the linear conductance across the edges can be written as $G\propto[2(J_2^{xy})^2+(J_2^z)^2]T^{K+1/K-2}$ by pertubation to second order. At low temperatures, the conductance can be obtained by substituting the normalized value of J_2 into the high temperature expression of G. As we show above, J_2 decreases near the fixed point with $J_1^{xy}=J_1^z=J_2=0$ and J_1 grows. If $1/2<K<1$, however, when the J_1 terms are of order 1, the J_2^{xy} term becomes relevant and grows. When the temperature decreases further, the strong coupling fixed point with $J_1^{xy}\to\infty$ is reached. Near this fixed point, J_2^{xy} and J_2^z are irrelevant and flow to zero. As a result, the conductance acquires a nontrivial temperature dependence. On the other hand, if $K<1/2$, the J_2 terms are always irrelevant and decrease to zero. Consequently, the conductance decreases monotonically as the temperature lowers. The temperature dependence of the linear conductance at different fixed points are shown in the Fig. 1(b).

We show above that a quantum dot coupled to two helical edge states of 2D topological insulators is described by a Kondo Hamiltonian. Weak repulsive interaction with $K<1$ on the edges, drives the system to a two-channel fixed point at low temperatures. At the fixed point, the residual impurity entropy is $\ln\sqrt{2K}$. The impurity specific heat and the conductance across the two edges through the quantum dot are also studied.

We thank Y. Avishai, C. H. Chung, D. Feldman and especially C. L. Kane for inspiring discussion and useful suggestions. KTL is supported by IAS-HKUST. P.A.L. acknowledge the support of NSF Grant No. DMR0804040 and the hospitality of IAS-HKUST.

[1] D. L. Cox and A. Zawadowski, Adv. Phys. **47**, 599 (1998) and references therein.
[2] P. Noziere and A. Blandin, J. Phys. **41**, 193 (1980).
[3] R. M. Potok *et al.*, Nature (London) **446**, 167 (2007).
[4] Y. Oreg and D. Goldhaber-Gordon, Phys. Rev. Lett. **90**, 136602 (2003).
[5] M. Fabrizio and A. O. Gogolin, Phys. Rev. B **51**, 17827 (1995).
[6] E. H. Kim, arXiv:cond-mat/0106575 (unpublished).
[7] V. R. Chandra *et al.*, Phys. Rev. B **75**, 045435 (2007).
[8] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).
[9] B. A. Bernevig *et al.*, Science **314**, 1757 (2006).
[10] L. Fu *et al.*, Phys. Rev. Lett. **98**, 106803 (2007).
[11] M. Konig *et al.*, Science **318**, 766 (2007).
[12] D. Hsieh *et al.*, Science **323**, 919 (2009).
[13] C. Wu *et al.*, Phys. Rev. Lett. **96**, 106401 (2006).
[14] A. Schiller and K. Ingersent, Phys. Rev. B **51**, 4676 (1995).
[15] J. Maciejko *et al.*, Phys. Rev. Lett. **102**, 256803 (2009).
[16] L. I. Glazman and M. E. Raikh, JETP Lett. **47**, 452 (1988).
[17] T. K. Ng and P. A. Lee, Phys. Rev. Lett. **61**, 1768 (1988).
[18] C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).
[19] A. O. Gogolin *et al.*, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, 1998).
[20] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
[21] V. J. Emery and S. Kivelson, Phys. Rev. B **46**, 10812 (1992).
[22] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).
[23] M. Fabrizio and A. O. Gogolin, Phys. Rev. B **50**, 17732 (1994).
[24] A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
[25] I. Affleck and A. W. W. Ludwig, Phys. Rev. Lett. **67**, 161 (1991).
[26] In the Refs. 21 and 23, the sine-Gordon Hamiltonian are written in terms of chiral bosonic fields. The bosonic fields in Eq. (3) are nonchiral.
[27] P. Fendley *et al.*, J. Stat. Phys. **79**, 799 (1995).
[28] H. Saleur, *Topological Aspects of Low Dimensional Systems*, edited by A. Comtet, *et al.* (Les Houches 69, Springer, 1999).