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CP phases of neutrino mixing in a supersymmetric $B-L$ gauge model with T_7 lepton flavor symmetry

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In a recently proposed renormalizable model of neutrino mixing using the non-Abelian discrete symmetry T_7 in the context of a supersymmetric extension of the standard model with gauged $U(1)_{B-L}$, a correlation was obtained between θ_{13} and θ_{23} in the case where all four parameters are real. Here we consider one parameter to be complex, thus allowing for one Dirac *CP* phase δ_{CP} and two Majorana *CP* phases $\alpha_{1,2}$. We find a slight modification to this correlation as a function of δ_{CP}. For a given set of input values of Δm^2_{21}, Δm^2_{32}, θ_{12}, and θ_{13}, we obtain $\sin^2 2\theta_{23}$ and m_{ee} (the effective Majorana neutrino mass in neutrinoless double beta decay) as functions of $\tan\delta_{CP}$. We find that the structure of this model always yields small $|\tan\delta_{CP}|$.

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The most general 3×3 Majorana neutrino mass matrix has six complex entries, i.e. 12 parameters. Three are overall phases of the mass eigenstates which are unobservable. The nine others are three masses, three mixing angles, and three phases: one Dirac phase δ_{CP}, i.e. the analog of the one complex phase of the 3×3 quark mixing matrix, and two relative Majorana phases $\alpha_{1,2}$ for two of the three mass eigenstates. The existence of nonzero δ_{CP} or $\alpha_{1,2}$ means that *CP* conservation is violated. It is one of the most important issues of neutrino physics yet to be explored experimentally.

The application of the non-Abelian discrete symmetry A_4 [1] (and others) to neutrino mixing has been successful in explaining tribimaximal mixing, i.e. $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, and $\theta_{13} = 0$. In particular, a generic three-parameter A_4 model [2] predicts all of the above with $\delta_{CP} = \alpha_{1,2} = 0$, leaving the three neutrino masses arbitrary. Recently, the first evidence that $\theta_{13} \neq 0$ has been published [3] by the T2K Collaboration, i.e.

$$0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34) \tag{1}$$

for $\delta_{CP} = 0$ and normal (inverted) hierarchy of neutrino masses. Slightly different but similar ranges are obtained for nonzero values of δ_{CP}. More recently, the Double Chooz Collaboration has also reported [4] a measurement of

$$\sin^2 2\theta_{13} = 0.086 \pm 0.041(\text{stat}) \pm 0.030(\text{syst}). \tag{2}$$

Their best fit is obtained by minimizing its χ^2 as a function of δ_{CP}. However, all δ_{CP} values are allowed within one standard deviation. One month ago, the first precise measurement of $\sin^2 2\theta_{13}$ was announced by the Daya Bay Collaboration [5]:

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \tag{3}$$

based only on a rate analysis, resulting in a 5.2σ effect. It has been followed by the announcement this month of the RENO Collaboration [6]:

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}), \tag{4}$$

again based only on a rate analysis, resulting in a 4.9σ effect.

To account for $\theta_{13} \neq 0$, the original A_4 proposal has to be modified [7]. Similarly, the original supersymmetric $B-L$ gauge model with T_7 lepton flavor symmetry [8] (which obtained tribimaximal mixing) has to be replaced as well [9]. In that latter paper, it is shown that a neutrino mass matrix with four parameters allow a nonzero θ_{13}. Assuming that all four parameters are real, thus requiring two conditions among the six observables, i.e. the three masses and three mixing angles, the prediction

$$\sin^2 2\theta_{23} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{13} \tag{5}$$

is obtained. This scenario applies of course only to the case $\delta_{CP} = \alpha_{1,2} = 0$. Here we consider instead the case where one parameter is complex. We then have five real parameters to describe the three masses, the three mixing angles, and the three phases. Given five inputs, we should then be able to predict the other four parameters.

The tetrahedral group A_4 (12 elements) is the smallest group with a real 3 representation. The Frobenius group T_7 (21 elements) is the smallest group with a pair of complex 3 and 3∗ representations. It is generated by

TABLE I. Character table of T_7.

Class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	χ_{3^*}
C_1	1	1	1	1	1	3	3
C_2	7	3	1	ω	ω^2	0	0
C_3	7	3	1	ω^2	ω	0	0
C_4	3	7	1	1	1	ξ	ξ^*
C_5	3	7	1	1	1	ξ^*	ξ

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{6}$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. The character table of T_7 (with $\xi = -1/2 + i\sqrt{7}/2$) is given in Table I.

The group multiplication rules of T_7 include

$$\underline{3} \times \underline{3} = \underline{3}^*(23, 31, 12) + \underline{3}^*(32, 13, 21) + \underline{3}(33, 11, 22), \tag{7}$$

$$\begin{aligned} \underline{3} \times \underline{3}^* = {} & \underline{3}(21^*, 32^*, 13^*) + \underline{3}^*(12^*, 23^*, 31^*) \\ & + \underline{1}(11^* + 22^* + 33^*) + \underline{1}'(11^* + \omega 22^* + \omega^2 33^*) \\ & + \underline{1}''(11^* + \omega^2 22^* + \omega 33^*). \end{aligned} \tag{8}$$

Note that $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants and $\underline{3} \times \underline{3} \times \underline{3}^*$ has one invariant.

We now follow Ref. [9] in deriving the neutrino mass matrix. Under T_7, let $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $i = 1, 2, 3$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, and $\Phi_i' = (\phi'^0, -\phi'^-)_i \sim \underline{3}^*$. The Yukawa couplings $L_i l_j^c \Phi_k'$ generate the charged-lepton mass matrix

$$\begin{aligned} M_l &= \begin{pmatrix} f_1 v_1' & f_2 v_1' & f_3 v_1' \\ f_1 v_2' & \omega^2 f_2 v_2' & \omega f_3 v_2' \\ f_1 v_3' & \omega f_2 v_3' & \omega^2 f_3 v_3' \end{pmatrix} \\ &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} v, \end{aligned} \tag{9}$$

if $v_1' = v_2' = v_3' = v'/\sqrt{3}$, as in the original A_4 proposal [1].

Let $\nu_i^c \sim \underline{3}^*$, then the Yukawa couplings $L_i \nu_j^c \Phi_k$ are allowed, with

$$M_D = f_D \begin{pmatrix} 0 & v_1 & 0 \\ 0 & 0 & v_2 \\ v_3 & 0 & 0 \end{pmatrix} = \frac{f_D v}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{10}$$

for $v_1 = v_2 = v_3 = v/\sqrt{3}$, which is necessary for consistency (from the bilinear $\Phi_i \Phi_i'$ term in the superpotential) since $v_1' = v_2' = v_3' = v'/\sqrt{3}$ has already been assumed for M_l. Note that Φ and Φ' have $B - L = 0$, and both are necessary because of supersymmetry. However, the analysis of neutrino mixing does not involve these extra supersymmetric partners.

Now add the neutral electroweak Higgs singlets $\chi_i \sim \underline{3}$ and $\eta_i \sim \underline{3}^*$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu_i^c \nu_j^c \chi_k$ and $\nu_i^c \nu_j^c \eta_k$ (which has to be symmetric in i, j). Note that $\chi_i^* \sim \underline{3}^*$ is not the same as $\eta_i \sim \underline{3}^*$ because they have different $B - L$. This means that both $B - L$ and the complexity of the $\underline{3}$ and $\underline{3}^*$ representations in T_7 are required for this scenario. The heavy Majorana mass matrix for ν^c is then

$$\begin{aligned} M_{\nu^c} &= h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u_3' & u_2' \\ u_3' & 0 & u_1' \\ u_2' & u_1' & 0 \end{pmatrix} \\ &= \begin{pmatrix} A & C & B \\ C & D & C \\ B & C & D \end{pmatrix}, \end{aligned} \tag{11}$$

where $A = hu_2$, $B = h'u_2'$, $C = h'u_1' = h'u_3'$, and $D = hu_1 = hu_3$ have been assumed. This means that the residual symmetry in the singlet Higgs sector is Z_2, whereas in the doublet Higgs sector is Z_3. The necessary soft terms are completely listed in Ref. [9]. This choice allows nonzero θ_{13}, whereas the choice of Ref. [8] enforces $\theta_{13} = 0$.

The seesaw neutrino mass matrix is now

$$\begin{aligned} M_\nu &= -M_D M_{\nu^c}^{-1} M_D^T \\ &= \frac{-f_D^2 v^2}{3\det(M_{\nu^c})} \begin{pmatrix} AD - B^2 & C(B - A) & C(B - D) \\ C(B - A) & AD - C^2 & C^2 - BD \\ C(B - D) & C^2 - BD & D^2 - C^2 \end{pmatrix}, \end{aligned} \tag{12}$$

where $\det(M_{\nu^c}) = A(D^2 - C^2) + 2BC^2 - D(B^2 + C^2)$. Redefining the parameters A, B, C, D to absorb the overall constant, we obtain the following neutrino mass matrix in the tribimaximal basis:

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} D(A + D - 2B)/2 & C(2B - A - D)/\sqrt{2} & D(A - D)/2 \\ C(2B - A - D)/\sqrt{2} & AD - B^2 & C(D - A)/\sqrt{2} \\ D(A - D)/2 & C(D - A)/\sqrt{2} & (AD + D^2 + 2BD - 4C^2)/2 \end{pmatrix}. \tag{13}$$

This is obtained by first rotating with the 3×3 unitary matrix of Eq. (9), which converts it to the (e, μ, τ) basis, then by Eq. (14) below. Note that for $D = A$ and $C = 0$, this matrix becomes diagonal: $m_1 = A(A - B)$, $m_2 = A^2 - B^2$, $m_3 = A(A + B)$, which is the tribimaximal limit. Normal hierarchy of neutrino masses is obtained if $B \simeq A$ and inverted hierarchy is obtained if $B \simeq -2A$.

The neutrino mixing matrix U has 4 parameters: s_{12}, s_{23}, s_{13}, and δ_{CP} [10]. We choose the convention $U_{\tau 1}$, $U_{\tau 2}$, U_{e3}, $U_{\mu 3} \to -U_{\tau 1}, -U_{\tau 2}, -U_{e3}, -U_{\mu 3}$ to conform with that of the tribimaximal mixing matrix

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{14}$$

Then

$$\begin{aligned} \mathcal{M}_\nu^{(1,2,3)} &= \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix} \\ &= U_{TB}^T U \begin{pmatrix} e^{i\alpha_1} m_1' & 0 & 0 \\ 0 & e^{i\alpha_2} m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix} U^T U_{TB}, \end{aligned} \tag{15}$$

where $m'_{1,2,3}$ are the physical neutrino masses, with

$$m_2' = \sqrt{m_1'^2 + \Delta m_{21}^2}, \tag{16}$$

$$m_3' = \sqrt{m_1'^2 + \Delta m_{21}^2/2 + \Delta m_{32}^2} \text{(normal hierarchy)}, \tag{17}$$

$$m_3' = \sqrt{m_1'^2 + \Delta m_{21}^2/2 - \Delta m_{32}^2} \text{(inverted hierarchy)}. \tag{18}$$

If U and $\alpha_{1,2}$ are known, then all $m_{1,2,3,4,5,6}$ are functions only of m_1'.

In Ref. [9], the parameters A, B, C, D are assumed to be real, hence δ_{CP} and $\alpha_{1,2}$ are zero. We now consider $C = E + iF$ to be complex and A, B, D to be real. This means that $h'u_2'$ and $hu_1 = hu_3$ are real relative to hu_2, whereas $h'u_1' = h'u_3'$ is complex. Hence $u_1' = u_3'$ must be complex relative to u_2' for C to be complex and B to be real. This solution may be obtained with suitable choices of the complex parameters listed in Ref. [9]. Thus $m_{1,2,4}$ are real and $m_{3,5,6}$ are complex. Since $\mathcal{M}_\nu^{(1,2,3)}$ is in the tribimaximal basis, it can be diagonalized by an approximately diagonal unitary matrix. To first order, let

$$U_\epsilon = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}^* & 1 & \epsilon_{23} \\ -\epsilon_{13}^* & -\epsilon_{23}^* & 1 \end{pmatrix}, \tag{19}$$

then using

$$U_\epsilon \mathcal{M}_\nu^{(1,2,3)} U_\epsilon^T = \begin{pmatrix} e^{i\alpha_1'} m_1' & 0 & 0 \\ 0 & e^{i\alpha_2'} m_2' & 0 \\ 0 & 0 & e^{i\alpha_3'} m_3' \end{pmatrix}, \tag{20}$$

we obtain ϵ_{12}, ϵ_{13}, $\epsilon_{23,}$ and $\alpha'_{1,2,3}$ in terms of A, B, C, D, E, F Using the four measured values Δm_{21}^2, Δm_{32}^2, s_{12}, s_{13}, and varying δ_{CP}, we then obtain s_{23}, the physical relative Majorana phases $\alpha_{1,2}$ in Eq. (15), and the effective Majorana neutrino mass in neutrinoless double beta decay, i.e.

$$m_{ee} = |U_{e1}^2 e^{i\alpha_1} m_1' + U_{e2}^2 e^{i\alpha_2} m_2' + U_{e3}^2 m_3'|. \tag{21}$$

Because of the structure of Eq. (13) from the T_7 symmetry, even though the phase of the complex parameter C may be large, i.e. F/E large, $\tan\delta_{CP}$ cannot be too large, because in the limit $C = 0$, there can be no *CP* violation, so any *CP* violating effect has to be proportional to F/D where D sets the neutrino mass scale. This is typically less than one because $C \neq 0$ measures the deviation of $\tan^2\theta_{12}$ from the tribimaximal limit of $1/2$.

The unitary matrix $U' = U_{TB} U_\epsilon^T$ has entries

$$\begin{aligned} U_{e1}' &= \sqrt{\frac{2}{3}} + \sqrt{\frac{1}{3}}\epsilon_{12}, \qquad U_{e2}' = \sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}}\epsilon_{12}^*, \\ U_{e3}' &= -\sqrt{\frac{2}{3}}\epsilon_{13}^* - \sqrt{\frac{1}{3}}\epsilon_{23}^*, \end{aligned} \tag{22}$$

$$U_{\mu 3}' = -\frac{1}{\sqrt{2}} + \frac{\epsilon_{13}^*}{\sqrt{6}} - \frac{\epsilon_{23}^*}{\sqrt{3}}, \qquad U_{\tau 3}' = \frac{1}{\sqrt{2}} + \frac{\epsilon_{13}^*}{\sqrt{6}} - \frac{\epsilon_{23}^*}{\sqrt{3}}. \tag{23}$$

To obtain U, we rotate the phases of the μ and τ rows so that $U_{\mu 3}' e^{-i\alpha_3'/2}$ is real and negative, and $U_{\tau 3}' e^{-i\alpha_3'/2}$ is real and positive. These phases are absorbed by the μ and τ leptons and are unobservable. We then rotate the $\nu_{1,2}$ columns so that $U_{e1}' e^{-i\alpha_3'/2} = U_{e1} e^{i\alpha_1''/2}$ and $U_{e2}' e^{-i\alpha_3'/2} = U_{e2} e^{i\alpha_2''/2}$, where U_{e1} and U_{e2} are real and positive. The physical relative Majorana phases of $\nu_{1,2}$ are then $\alpha_{1,2} = \alpha_{1,2}' + \alpha_{1,2}''$. We now extract the three angles as well as δ_{CP} as follows:

$$\tan^2\theta_{12} = \left|\frac{U_{e1}'}{U_{e2}'}\right|^2 = \left(\frac{1}{2}\right) \frac{(1 - \sqrt{2}\mathrm{Re}(\epsilon_{12}))^2 + 2(\mathrm{Im}(\epsilon_{12}))^2}{(1 + \mathrm{Re}(\epsilon_{12})/\sqrt{2})^2 + (\mathrm{Im}(\epsilon_{12}))^2/2}, \tag{24}$$

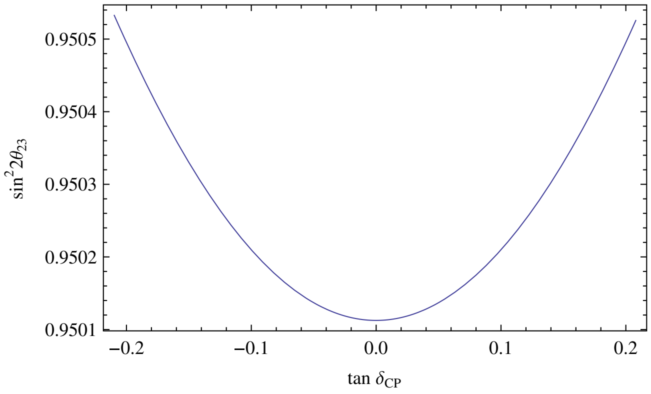

FIG. 1 (color online). $\sin^2 2\theta_{23}$ versus $\tan\delta_{\rm CP}$ for normal hierarchy.

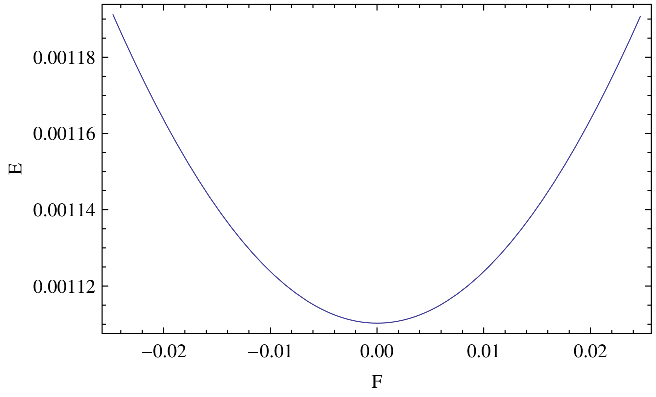

FIG. 3 (color online). T_7 complex parameters E versus F for normal hierarchy.

$$\tan^2\theta_{23} = \left|\frac{U'_{\mu 3}}{U'_{\tau 3}}\right|^2 = \frac{(1-(\mathrm{Re}(\epsilon_{13}-\sqrt{2}\epsilon_{23})/\sqrt{3})^2 + (\mathrm{Im}(\epsilon_{13}-\sqrt{2}\epsilon_{23}))^2/3}{(1+(\mathrm{Re}(\epsilon_{13}-\sqrt{2}\epsilon_{23})/\sqrt{3})^2 + (\mathrm{Im}(\epsilon_{13}-\sqrt{2}\epsilon_{23}))^2/3}, \qquad (25)$$

$$\sin\theta_{13}e^{-i\delta_{CP}} = U'_{e3}e^{-i\alpha'_3/2}. \qquad (26)$$

To see the approximate dependence of U_ϵ on the T_7 parameters A, B, D, E, F, we assume normal hierarchy and let

$$A = D + \delta_1, \qquad B = D + \delta_2, \qquad C = E + iF. \qquad (27)$$

Expanding in $\delta_{1,2}$, E, F over D, we then have

$$m_1 = \frac{D}{2}(\delta_1 - 2\delta_2), \qquad m_2 = D(\delta_1 - 2\delta_2) - \delta_2^2, \\ m_3 = 2D^2 + \frac{D}{2}(\delta_1 + 2\delta_2), \qquad (28)$$

$$m'_1 = m_1 - \frac{\delta_1^2}{8}, \qquad m'_2 = m_2, \qquad m'_3 = m_3, \qquad (29)$$

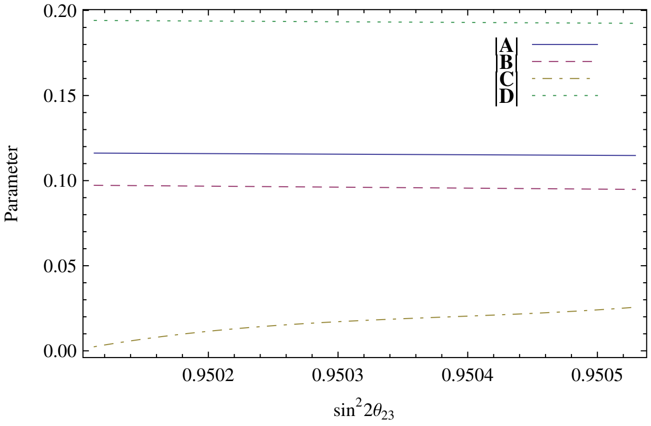

FIG. 2 (color online). T_7 parameters for normal hierarchy.

$$m_4 = \frac{D}{2}\delta_1, \qquad m_5 = -\frac{\delta_1}{\sqrt{2}}(E + iF), \\ m_6 = -\frac{E + iF}{\sqrt{2}}(\delta_1 - 2\delta_2), \qquad (30)$$

$$\mathrm{Re}(\epsilon_{12}) = \frac{\sqrt{2}E}{D}\left(1 - \frac{\delta_1^2}{4D(\delta_1 - 2\delta_2)}\right)\left(1 + \frac{\delta_1^2 - 8\delta_2^2}{4D(\delta_1 - 2\delta_2)}\right)^{-1}, \qquad (31)$$

$$\mathrm{Im}(\epsilon_{12}) = \frac{\sqrt{2}F}{3D}\left(1 - \frac{\delta_1^2}{4D(\delta_1 - 2\delta_2)}\right)\left(1 - \frac{\delta_1^2 + 8\delta_2^2}{12D(\delta_1 - 2\delta_2)}\right)^{-1}, \qquad (32)$$

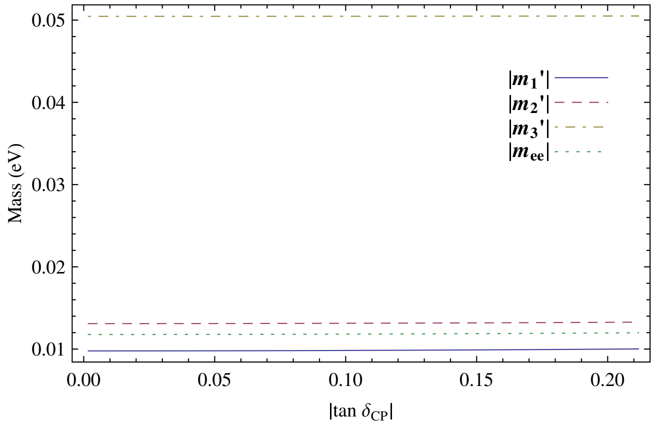

FIG. 4 (color online). Physical neutrino masses and the effective neutrino mass m_{ee} in neutrinoless double beta decay for normal hierarchy.

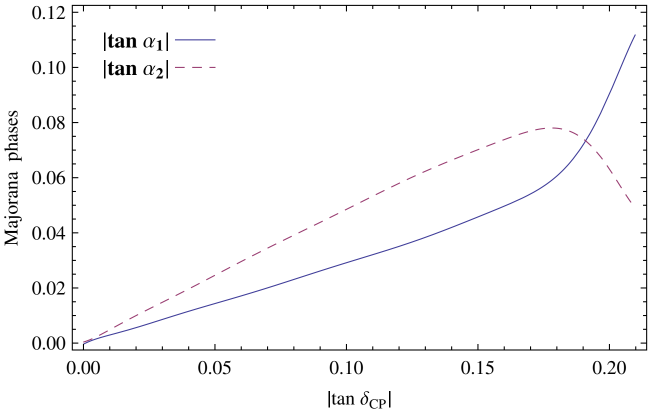

FIG. 5 (color online). Majorana phases $|\tan\alpha_1|$ and $|\tan\alpha_2|$ versus $|\tan\delta_{CP}|$ for normal hierarchy.

$$\epsilon_{13} = -\frac{\delta_1}{4D}\left(1+\frac{\delta_2}{D}\right)^{-1}, \qquad \epsilon_{23} = \frac{\delta_1}{2\sqrt{2}D^2}(E+iF). \tag{33}$$

Using Eq. (26), and neglecting α_3', we obtain

$$\tan\delta_{CP} = \frac{F}{D}\left(1+\frac{\delta_2}{D}\right)\left[1-\frac{E}{D}\left(1+\frac{\delta_2}{D}\right)\right]^{-1}. \tag{34}$$

Assuming inverted hierarchy, we let

$$A = D+\delta_1, \quad B = -2D+\delta_2, \quad C = E+iF, \tag{35}$$

then

$$\begin{aligned} m_1' &= m_1 = 3D^2 + \frac{D}{2}(\delta_1 - 2\delta_2), \\ m_2' &= m_2 = -3D^2 + D(\delta_1 + 4\delta_2), \end{aligned} \tag{36}$$

$$m_3' = m_3 = -D^2 + \frac{D}{2}(\delta_1 + 2\delta_2), \tag{37}$$

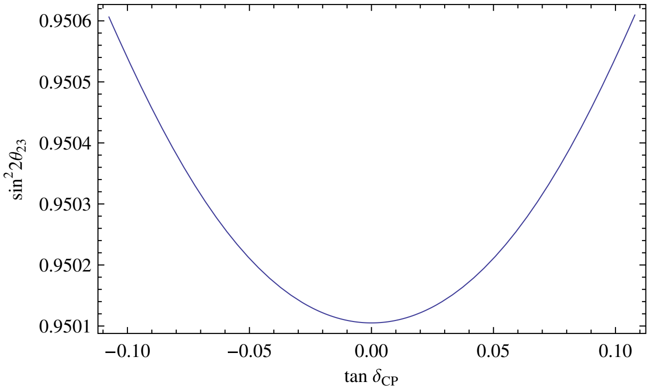

FIG. 6 (color online). $\sin^2 2\theta_{23}$ versus $\tan\delta_{CP}$ for inverted hierarchy.

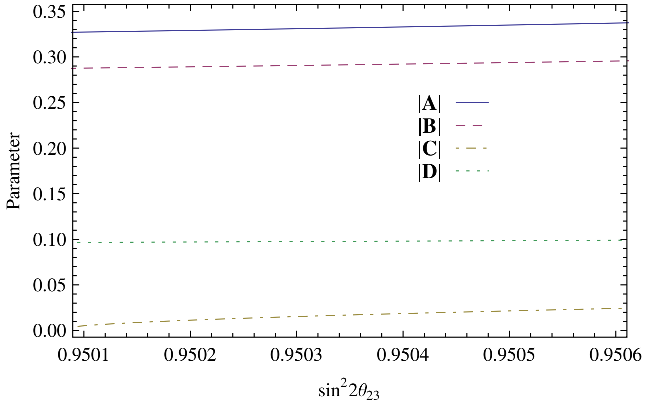

FIG. 7 (color online). T_7 parameters for inverted hierarchy.

$$\begin{aligned} m_4 &= \frac{D}{2}\delta_1, \qquad m_5 = -\frac{\delta_1}{\sqrt{2}}(E+iF), \\ m_6 &= -\frac{E+iF}{\sqrt{2}}(6D+\delta_1-2\delta_2), \end{aligned} \tag{38}$$

$$\mathrm{Re}\,(\epsilon_{12}) = -\frac{E}{D\sqrt{2}}\left(1+\frac{\delta_1}{6D}-\frac{\delta_2}{3D}\right)\left(1-\frac{\delta_1}{12D}-\frac{5\delta_2}{6D}\right)^{-1}, \tag{39}$$

$$\mathrm{Im}\,(\epsilon_{12}) = \frac{2\sqrt{2}F}{\delta_1+2\delta_2}\left(1+\frac{\delta_1}{6D}-\frac{\delta_2}{3D}\right), \tag{40}$$

$$\mathrm{Re}\,(\epsilon_{13}) = \frac{\delta_1}{8D}\left(1-\frac{\delta_2}{2D}\right)^{-1}, \qquad \mathrm{Im}(\epsilon_{13}) = -\frac{F\delta_1}{16\sqrt{2}D^2}, \tag{41}$$

and ϵ_{23} is determined by

$$\begin{aligned} -\epsilon_{23}^* m_2 + \epsilon_{23} m_3 = {} & -m_5 + \epsilon_{12}^* m_4 + \epsilon_{13}^* m_6 \\ & - \epsilon_{13}^*\epsilon_{12}^* m_1. \end{aligned} \tag{42}$$

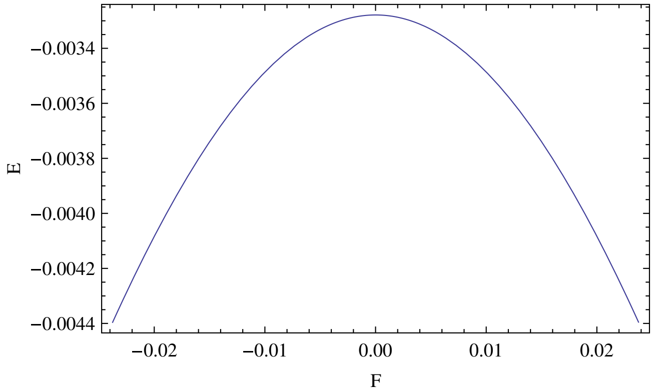

FIG. 8 (color online). T_7 complex parameters E versus F for inverted hierarchy.

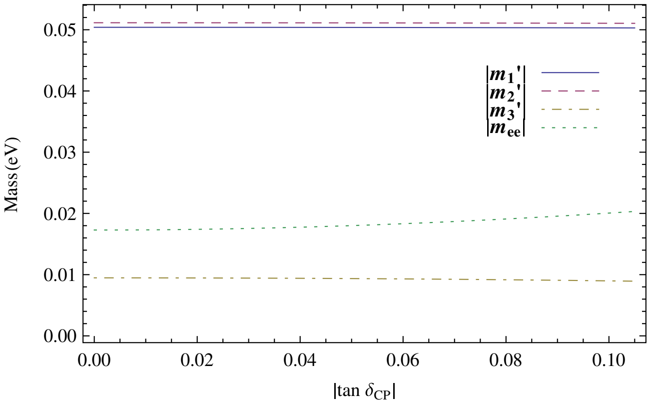

FIG. 9 (color online). Physical neutrino masses and the effective neutrino mass m_{ee} in neutrinoless double beta decay for inverted hierarchy.

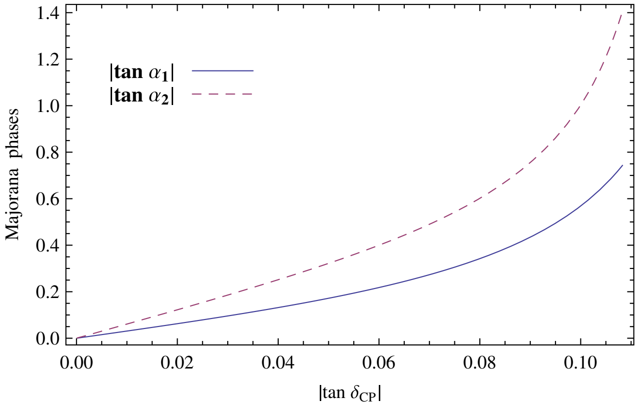

FIG. 10 (color online). Majorana phases $|\tan\alpha_1|$ and $|\tan\alpha_2|$ versus $|\tan\delta_{\rm CP}|$ for inverted hierarchy.

For our numerical analysis, we set

$$\Delta m^2_{21} = 7.59 \times 10^{-5}\ \text{eV}^2, \qquad \Delta m^2_{32} = 2.45 \times 10^{-3}\ \text{eV}^2, \tag{43}$$

$$\sin^2 2\theta_{12} = 0.87, \qquad \sin^2 2\theta_{13} = 0.092. \tag{44}$$

We then diagonalize Eq. (15) exactly and scan for solutions satisfying the above experimental inputs. Assuming normal hierarchy, we find $\sin^2 2\theta_{23}$ to range from 0.9501 for $\delta_{CP} = 0$ to 0.9505 for $|\tan\delta_{CP}| = 0.2$, as shown in Fig. 1. This is an imperceptible change, so our model prediction for $\sin^2 2\theta_{23}$ is basically unchanged from the real case. We show the absolute values $|A|$, $|B|$, $|D|$, and $|C|$ as functions of $\sin^2 2\theta_{23}$ in Fig. 2, and E versus F in Fig. 3. As expected, F/E may be large, but $\tan\delta_{CP} \simeq F/D$ remains small. We then plot the three physical neutrino masses $m'_{1,2,3}$ as well as m_{ee} as functions of $|\tan\delta_{CP}|$ in Fig. 4, and the Majorana phases $\alpha_{1,2}$ versus $|\tan\delta_{CP}|$ in Fig. 5. For inverted hierarchy, we show in Figs. 6–10 the corresponding plots. We note again that $\tan\delta_{CP}$ is small, but now $\alpha_{1,2}$ are much larger. This can be seen from Eq. (40) versus Eq. (32).

In conclusion, we have studied how the T_7 model of Ref. [9] allows CP violation in the neutrino mixing matrix. We assume three real parameters A, B, D and one complex parameter $C = E + iF$, from which nine physical observables may be derived. Given the experimental inputs Δm^2_{21}, Δm^2_{32}, $\sin^2 2\theta_{12}$, and the recently measured $\sin^2 2\theta_{13}$, the remaining five observables depend on only one variable which we choose to be δ_{CP}. Because of the structure of the neutrino mass matrix constrained by T_7, even if C has a large phase, i.e. F/E is large, $\tan\delta_{CP}$ remains small. For $\sin^2 2\theta_{13} = 0.092$ and $\sin^2 2\theta_{12} = 0.87$, we find $\sin^2 2\theta_{23}$ to be essentially fixed at 0.95 as $|\tan\delta_{CP}|$ changes from 0.0 to 0.2. The Majorana phases $\alpha_{1,2}$ are comparable to δ_{CP} in magnitude for normal hierarchy, but are much larger for inverted hierarchy. We have not studied the more general case where all parameters are allowed to be complex.

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